Informal and absolute provability: from Kreisel and Gödel to Prawitz and Girard

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We aim at investigating whether the basic tenets of two historically relevant theories, Kreisel's informal rigour and Gödel's absolute provability, can match those of existing formal semantics. The theories with respect to which we evaluate Kreisel's and Gödel's ideas are Prawitz's proof-based semantics in terms of arguments and grounds, and Girard's proof-nets and Ludics. Prawitz's provability, because of its semantic character and of Gödel's incompleteness, is not reducible to the concept of derivability in a formal system. Therefore, it could make sense to ask whether Prawitz's provability may be qualified as informal in the sense of Kreisel - i.e. not embeddable in a recursive system, although exemplified by some such systems - or as absolute in the sense of Gödel - i.e. not relative to any recursive system, either in the sense of being inexhaustibly more powerful than any such system, or in the sense of being universally applicable to any such system. We argue that Prawitz's view might cope with Kreisel's one, while Gödel seems to undertake a different standpoint, reminding of Girard's proof-nets and Ludics. Indeed, in a stronger reading Gödel's absoluteness may be understood as aiming at a concept of untyped proof where only "positional" or "geometrical" or "dynamic" aspects matter. This is in line with Girard's idea of analysing proofs as oriented graphs, or as designs where formulas are replaced by numerical addresses standing for positions occupied by formulas. In both cases, Girard's approach allows for an untyped reading, where the typing (and hence the properties) of a proof depend on global and "geometrical" features of an underlying structure.