Prof. Husserl legt dem Besucher die Vorzüge der Phänomenologie als wissenschaftlicher Philosophie dar

Husserl
on
Formal Mathematics
and
how it relates to intuition

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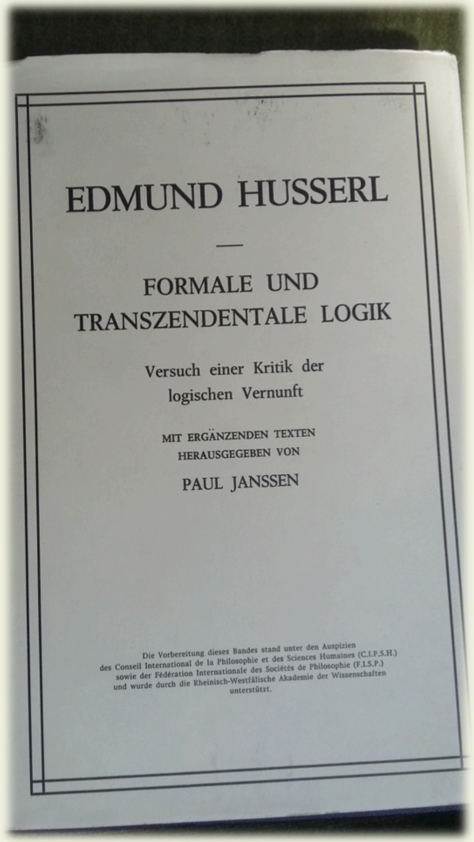
1. Husserl on formal mathematics

Formale und transzendentale Logik, 1929

-His ”most mature, if too concentrated” work; contains a “*definitive clarification of the sense of pure formal mathematics* …, according to the prevailing intention of mathematicians” (FTL, 11).

-The “prevailing intention of mathematicians” > “mathematics of mathematicians” vs. “mathematics with a logical interest”

-Zermelo vs. Hilbert, Weyl



- In *Prolegomena* (1900) Leibnizian *mathesis unversalis,* partially realized in Riemann's theory of manifolds, Grassmann's theory of extensions, Hamilton's quaternions, and Cantor's set theory (e.g., §§60, 68, 70)
- In FTL, the core idea remains unchanged: ”The great advance of modern mathematics, particularly as developed by Riemann and his successors, consists … in its having gone on to view *such system-forms themselves as mathematical Objects*, to alter them freely, universalize them mathematically, and particularize the universalities – … in conformity with the superordinations and subordinations that present themselves in the province of the formal” (§30).
- I.e., modern structural mathematics.
- Combination of ”formal apophantics” (theory of judgments) and ”formal ontology” (study of formal objects, i.e., mathematics).
- Acts of judgments include not only acts of predicating something of the objects, but also acts of collecting, counting, ordering and combining mathematically (FTL, §§39; 100)
- Theory of judgments embraces all of pure mathematics

  Husserl explicitly points out that it includes traditional analysis [die traditionelle formale “Analysis” der Mathematiker], set theory [die Mathematik der Mengen], theories of combinations and permutations, cardinals or ordinals belonging to various levels, of manifolds [Mannigfaltigkeiten], etc. (FTL, §24)

2. Mathematics of mathematicians and mathematics with logical interest

§51: “the mathematician as such need not be at all concerned with the fact that there actually are multiplicities in concrete ‘actuality’.“

§52: “Mathesis pura” as properly logical and as extra-logical. The “mathematics of mathematicians”

“one can set up a whole science that, freed from the specifically logical aim, neither explores nor intends to explore anything beyond the universal realm of pure apophantic senses. It becomes apparent that, when questions about possible truth are consistently excluded in this manner, and the truth-concept itself is similarly excluded, one has not actually lost any of this logical mathesis; one still has the whole of it: as *‘purely’ formal mathematics’*. ”

“One must see that a formal mathematics, reduced to the above described purity, has its own legitimacy and that, for mathematics there is in any case no necessity to go beyond that purity. … in this manner the *proper sense of ‘formal mathematics’,* the mathematics to which every properly logical intention (that is: every intention to a theory of science) remains alien – the mathematics of mathematicians – at last becomes *fundamentally clarified*.”

3. Kinds of evidence: distinctness [Deutlichkeit] vs. clarity [Klarheit]

- Difference between the two is ultimately that mathematics of mathematicians and with logical interest aim at different kinds of evidence

  Mathematics (of mathematicians) > distinctness [*Deutlichkeit*] – logic of non-contradiction

  Mathematics with logical interest > distinctness and clarity [*Klarheit*] – logic of truth

  (In addition, both seek grammatical rigor. Husserl also talks about evidence related to “a new sort of categorial formation,” namely, constructional infinities > openness to further kinds of evidence to surface in mathematics.)

- §53, Husserl’s example: Euclidian geometry *as a possible system of true propositions* vs. or as one among an open infinity of possible deductive sciences having this same categorial form, i.e., as a system of propositions purely as senses, in distinct evidence, as a systematic whole, “unifiable without contradiction”
- The distinction between the two kinds of evidence is first presented in the lecture course *Erste Philosophie* (1923-24)
- Purely mathematical theories have “unity of an internally coherent validity” (Husserl 1956, 19/20).
- Whether a judgment belongs to this theoretical unity is independent of the question of whether the judgments are true or false.

- Distinctness derives from the experienced “theoretical unity”; whereas clarity is paradigmatically obtained by seeing that what is judged is indeed the case. I.e., “originarily”, “in person”.
    - “Judging with ‘clarity’ has at once clarity of the affairs, in the performance of the judgment-steps, and clarity of the predicatively formed affair-complex in the whole judging” … “it is a new evidence, pertaining to a givenness originaliter of the affairs themselves, of the predicatively formed affair-complex itself, at which one aims in the judging that strives toward cognition”… (§16b).
- Taking stock: there is purely formal mathematics, characterized by the evidence of distinctness, and applied mathematics (logic of truth) that is determined by the evidence of clarity (i.e., what is actually true). i.e., two kinds of “intuition”, where in the case of clarity, the notion of intuition is paradigmatically perception of the objects in the world.

4. The Transitional Link

“we require here an important supplementation of the pure logic of non-contradiction, a supplementation that, to be sure, goes beyond formal mathematics proper, but still does not belong to truth-logic. It is a matter, so to speak, of a *transitional link [Übergangsglied]* between them”(§82).

- Aim is to show how judgments of formal mathematics ultimately relate to judgments about individuals.

“For *mathesis universalis*, as formal mathematics, these ultimates have no particular interest. Quite the contrary for *truth-logic*: because ultimate substrate-objects are *individuals*, about which very much can be said in formal truth, and *back to which all truth ultimately relates*“ (§82).

- Whereas pure mathematicians need not care about it, the mathematician with a logical (= foundational) interest wants to know how mathematics relates to judgments about individual objects, and this is what the “transitional link” shows.

Transitional link:

Husserl insists on the traditional form of judgement

S is p

The forms of judgment leave it indeterminate whether the terms are complex or not (i.e., in Husserl's terms, whether the subject or the predicate are syntactically structured).

> ”But it can be seen a priori that *any actual or possible judgment leads back to ultimate cores* when we follow up its syntaxes; accordingly that it is a syntactical structure built ultimately, though perhaps far from immediately, out of *elementary cores, which no longer contain any syntaxes”* (§82).

”by reduction, we reach a corresponding ultimate, that is: *ultimate substrates*- … *absolute subjects* …, *ultimate predicates*, … *ultimate universalities*, *ultimate relations*” …

”the reduction signifies that, *purely by following up the meanings*, we reach *ultimate something-meanings*; first of all, then, as regards the meant or supposed judgment-objects, *supposed absolute objects-about-which*.”

If we operate only with the distinct judgment-senses, one can only reach a claim about ”sense-elements” as the ultimate ”core-stuffs.”

§83: In logic of truth, there is a corresponding reduction to truths, to judgments that are about individual objects,

> ”objects that therefore contain within themselves no judgment-syntaxes and that, in their experienceable factual being, are prior to all judging”… ”reductive deliberation teaches, as an *Apriori,* that *every conceivable judgment ultimately … has relation to individual objects* (in an extremebly broad sense, real objects), and therefore has *relation to a real universe*, a ’world’ or a ’*world-province*,’ for which it holds good”.

- The ”transitional link” is supposed to reveal ”hidden intentional implications” in judging. It uncovers ”the sense-genesis” of judgments. (§85)
- It shows that the lowest level is judgements about individuals;

  ”and consequently, in the case of evident judgments, in the sense of seeings of the predicatively formed affair-complexes themselves, it brings us to those evidences of something individual that belong to the simplest type. These are the pure and simple experiential judgments, judgments about data of possible perception and memory, which give norms for the correctness of categorical judicial meanings at the lowest level concerning individuals” (§86)

So, like for Hilbert and for Weyl, for Husserl, logic presupposes some pre-existing individual objects, hence Husserl’s ”foundations” closer to them than to Brouwer.

5. Abstraction principles or intuitionistic type theory?

- Recently, Husserl’s transitional link has been described in terms of (Fregean) abstraction principles (Costantini).
- Dynamic abstractionism (Linnebo) to introduce new objects.
- The account regards the reduction in the transitional link as a ”denominalization”, where ”nominalization” is a linguistic counterpart of the process of abstraction (that generates new objects), so that the truth of complex judgements is *grounded* on the objects of our experience.
- Asymmetrical abstraction principles to expand domains; PFOL with a modalized quantifier $\Box\forall$ to express general statements over any possible expansion. (E.g., in *Ideas I*, §119, Husserl talks about ”plural consciousness” and the law of nominalization)

An interesting suggestion, but…

- The approach is untyped, whereas Husserl's universe is typed, as indicated by his insistence on the traditional form of judgment;
- The way Husserl's theory of judgment and mathematics are ”entangled” shows a tighter connection between ”logic” and mathematics.
- Husserl seems to suggest that the reduction is mechanical, i.e., it is computable (this would ensure the mediation of evidence).
- In sum, the transitional link has properties of intuitionistic type theory. (Typed universe, the form of judgment, entanglement due to Curry-Howard isomorphism, and due to it, decidability.)
- Be that as it may, in Husserl's view the foundational interest strives us to show how mathematics is related to a perception of ordinary objects and judgments about them.

What about the mathematics of mathematicians?

- Husserl explains how when we ascend from given individual objects to the formal Apriori, ”each *’individual’* must be *emptied to become anything whatever*” (§86). I.e., ”In place of something individual, there enters everywhere the positing of ’a certain substrate (of whatever sort) about which one can judge”(§86).
- ”*The evidence of laws pertaining to the analytic Apriori* needs no such intuitions of determinate individuals” (§86).
- ”S is p” > ”p”; loss of ”determinateness” of individuals, i.e., the typification.
- The evidence of distinctness, i.e., non-contradictoriness of the unity of judgment, as he talked about it in *Erste Philosophie* lectures in 1923-4; > ”existence of a model”?

But, here:

- ”Nevertheless, the sense-relation of all categorial meanings to something individual, … surely *cannot be insignificant for the sense and the possible evidence of the laws of analytics*, including the highest ones, the principles of logic. Otherwise, how could those laws claim formal-ontological validity: validity for *everything conceivably existing*?” (§86).

The precise source of distinctness:

- ”the unitary effectibility of the judgment-content”, or the ideal ’existence’ of the judgment-content”.
- Husserl: ”the possibility of a judgment (as a meaning) is rooted not only in the syntactical forms but also in the syntactical stuffs.” (§89b)
- The intentional genesis: ”Every judgment as such has its *intentional genesis* or, as we can also say, its essentially necessary *motivational foundations*, without which it could not at first exist in the primitive mode, certainty, nor be modalized thereafter. These foundations include the necessity that the *syntactical stuffs* occuring in the unity of a judgment *have something to do with one another*” (§89b).
- ”*Prior* to all judging, there is a universal experiential basis. It is always presupposed *as a harmonious unity of possible experience*. In this harmony, everything has ’to do’ materially with everything else…. Thus, *in respect of its content, every original judging* and every judging that proceeds coherently, has *coherence by virtue of the coherence of the matters in the synthetic unity of the experience*, which is the basis on which the judging stands. We do not intend to say in advance that there can be only one universe of possible experience as the basis for judgment, and that therefore every intuitive judgment has the same basis and all judgments belong to a single materially coherent whole. To reach a decision about that would require a separate investigation.” (§89b).

- Husserl seems to be saying that in formal mathematics the information about the ”determination” of individuals is abstracted away.
- However, it is ultimately needed to show that the individuals can ”materially” relate to each other, in the harmonious unity of experiential basis.
- Coherence is not a merely syntactic matter (as opposed to Hilbert).
- Semantic? In terms of the existence of models (as I have suggested before)? – not quite, but perhaps in terms of meaning explanations? Syntactical stuffs as the ”typification” or ”computable content” to be included in the meaning genesis?

- But, this seems to contradict Husserl’s claim that the pure mathematician needs not to care about such foundational matters.
- Either Husserl holds that even pure mathematics has to be restricted to be so founded (Husserl would be unaware of any restrictions), or else he introduces another sense in which mathematics may have foundations, in distinct evidence.
- My view: Husserl is open to new forms of evidence; if he lived now, he should and would acknowledge that this is not the evidence according to the *prevailing intentions* of pure mathematicians. It does carve out a ”distinctly evident” part of mathematics.
- Phenomenology, in general, is a metaphysically neutral method with which any experience can be examined. No reason to cut out, say, descriptive set theory.
- Foundational interests of mathematicians may suggest the existence of further kinds of evidence that carve out some other parts of mathematics.

7. Conclusion: ”Mathematics of mathematicians” and mathematics with two kinds of foundational interests:

- Formal mathematics can be approached in two ways: purely mathematically and ”logically,” with a foundational interest.
- Husserl claims that the former seeks evidence of distinctness that ultimately derives from the harmonious unity of experience, in which the somethings-whatever form a ”material” unity.
- The latter seeks evidence of clarity, which is paradigmatically obtained by perceiving an object.
- These kinds of evidence are reached with the ”transitional link”
- Husserl claims that even though the pure mathematician needs not to care about it, even her subject matter is ultimately related to the harmonious unity of experience via the transitional link.
- Husserl’s view of distinct evidence cannot cover all of ”mathematics of the mathematicians”. A suggestion: Today, phenomenologist should clarify new, contemporary kinds of evidence that determine mathematicians intentions today.

Thank you!

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