

Mathematical Diagrams: A Kaplanian Account?

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Outline

1. Introduction

2. Identity of words

1. Identity and the type/token distinction
2. Identity and transmission (Kaplan)

3. Identity of diagrams

1. Transmission of texts, transmission of diagrams
2. Copying and printing

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Introduction

Shin 2012 criticizes a view she calls “complacent dualism”:

- Diagrams are more useful for discovering a solution than symbols are.
- Symbols are more useful for constructing a formal proof than diagrams are.

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	Brainstorming	Proofs
Diagrammatic representation	+	-
Symbolic representation	-	+

According to Shin, the complacent view, widely shared in the literature, is false and lazy.

1. Lazy: it does not explain why diagram is better than symbol in brainstorming, and why symbol is better than diagram in formal proof.
2. False: diagrammatic representations are not less useful than symbolic representations for constructing formal proofs.

In this talk, we will oppose both the complacent view and the alternative view that Shin develops.

As we will show in **section 1**, both views are based on the type/token identity claim for symbol:

[t/tic] The identity of a symbolic token is determined by the type it belongs to.

In **section 2**, we will present an alternative conception of symbols, that has been developed by Kaplan. According to Kaplan, [t/tic] should be replaced by the transmissibility identity claim:

[tic] The identity of a symbolic token is determined by the chain of transmission it belongs to.

This development has apparently nothing to do with diagrams. But in **sections 3 and 4**, we will try to extend Kaplan's view (whose main core is to link the identity of a representation to its chain of transmission within a community) to diagrams.

More specifically, we will argue that :

- **Section 3:** Kaplan's view provides us with the right framework to accommodate the recent historical works done on the transmission of the Euclidean diagrams through the ages.
- **Section 4:** Kaplan's idea that the spread of printing techniques is a turning point in the history of symbolic transmission extends well to the history of the transmission of Euclidean diagrams .

Identity and the Type/Token Distinction

Let me begin with the question the complacent view has to address to overcome its laziness:

What makes diagrams so unsuitable for expressing rigorous reasoning?

Shin: “A main concern related to diagrammatic reasoning is that we mistakenly rely on properties of a **specific token** of a figure and make an incorrect inference about a **more general type** of case. ... Let me call this specific concern the **token fallacy** problem. Many would agree that the **token fallacy problem** is the main reason why diagrams have not been welcomed as part of formal proofs.”

Zinedine Zidane, *Zinedine Zidane*, ZINEDINE ZIDANE, ZINEDINE ZIDANE, ...

☐ Four tokens of the same Zinedine Zidane type

Mastering a language means (among other things) being able to extract without any difficulty the general type under the particular tokens.

Zinedine Zidane, *Zinedine Zidane*, ZINEDINE ZIDANE, **Sébastien Gandon**,
ZINEDINE ZIDANE, ...

Look for the intruder!

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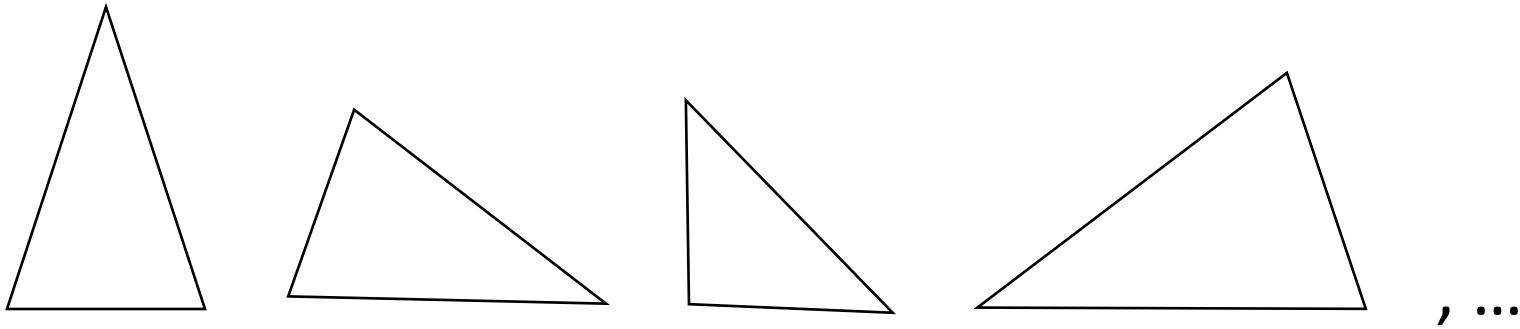
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Proclus on Euclid's proof: enunciation, setting-out, specification, construction, proof (proper), and conclusion.

Enunciation and conclusion are general. But the intermediate steps are particular.

Shin: "The crux of the ... problem is "How can we make a legitimate move from [the intermediate steps] above to the conclusion of the main proof?" A quick answer is "By not relying on any accidental features of particular figures, but by using only their general features." In other words, **"Don't commit the token fallacy."**



What type of tokens do these tokens refer to?

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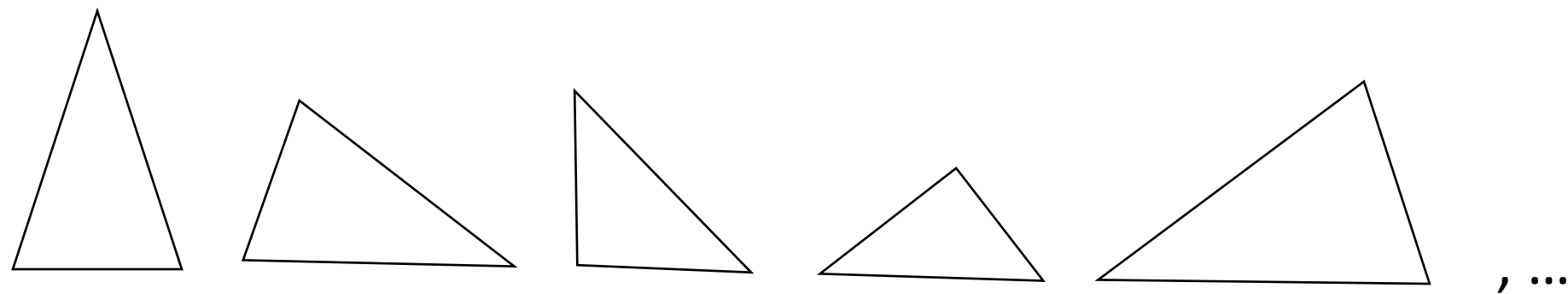
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Shin: “A main concern related to diagrammatic reasoning is that we mistakenly rely on properties of a **specific token** of a figure and make an incorrect inference about a **more general type** of case.”

In other words, the **token fallacy** seems to be the reason behind the complacent view: the use of diagrammatic representations in proof require particular care, which is not the case for the symbolic representations, where tokens are immediately referred to their type. Token fallacy seems then intrinsically connected to the use of diagrams.

Shin: “A main concern related to diagrammatic reasoning is that we mistakenly rely on properties of a **specific token** of a figure and make an incorrect inference about a **more general type** of case.”

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But, according to Shin, one must resist this conclusion.

The ability of extracting the type from symbolic tokens is made up of two main parts:

- The capacity to identify the tokens of letters composing the words
- The capacity to identify the types of the component letters

To extend this ability to diagrammatic tokens, one should be able:

- To identify the basic components of a diagram
- To identify the types of the basic diagrammatic components

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The second part does not pose any difficulty, according to Shin. It is the first part which is tricky: most of the time, there is **no single way** to break down a diagram into its basic components.

Shin: “*Gestalt* phenomena take place in diagrammatic representation more often than in symbolic representation mainly because one is spatial and the other is linear. More importantly, in many cases symbolic systems have to adopt unique readability to block ambiguity. ... On the other hand, diagrams are spatial and **multiple readings are almost inevitable**. A new way to carve up the domain, thanks to new particulars, helps us to see more easily what is implied in our premises.”

Why do diagrams, more than symbols, fall prey to the token fallacy ?

Shin's diagnosis: because of the fact that there are always multiple ways to decompose them.

And this diagnosis paves the way to a cure: formalization.

Unique readability is not at all a prerequisite for formalization. On the contrary, formalization can provide us with the means to distinguish the various decompositions of the same diagram, and thus with the means to avoid the token fallacy.

Shin: "Generalization, without committing the token fallacy and without sacrificing accuracy, could be achieved by formalization, and symbolization is not the only way to obtain formal systems."

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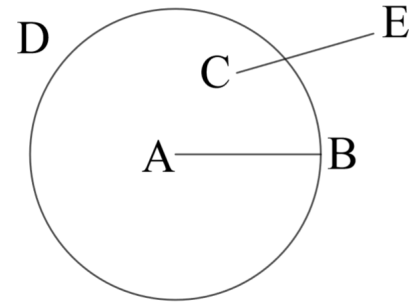
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This insight has first been developed in Barwise & Etchemendy 1990. It is used in Shin (for instance in 2002) and Mumma (for instance in 2010) to show that diagrammatic reasoning can be used for constructing formal proof (against the complacent view).

“Consider the diagram



Many distinct constructions could have produced it. For instance, the initial configuration could have been the segment AB, and the construction steps leading to the diagram could have been:

- draw the circle D with center A and radius AB.
- pick a point C in the circle D, and a point E outside it.
- produce the ray CE from the point C.

Call this **construction C1**. Alternatively, it is possible that the initial configuration consists of the segment AB and the points C and E, while the construction consists of the following two steps:

- draw the circle D with center A and radius AB.
- produce the ray CE from the point C

Call this **construction C2**.”

(Mumma, 2008, 230)

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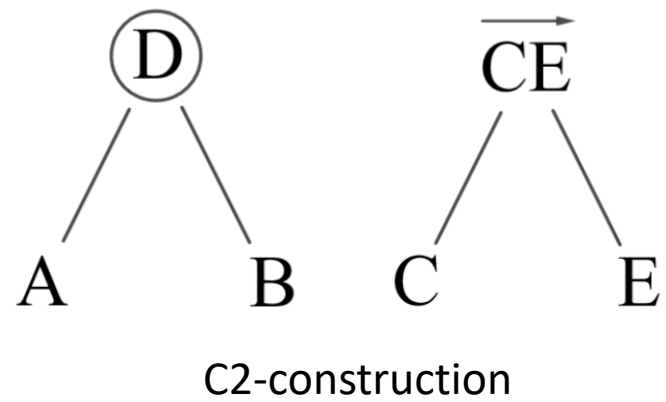
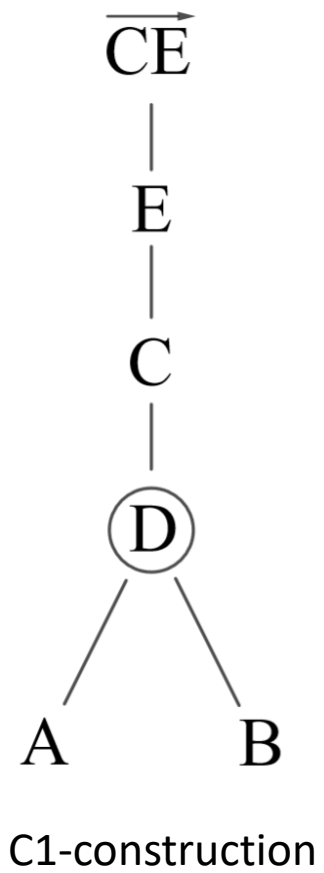
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The token fallacy does not show that [t/tic] cannot be extended to diagram, but that complex diagrams do not obey unique readability. [t/tic] holds for simple diagrammatic components, and it is possible to extend [t/tic] to diagrams, once unique readability is given up.

	[t/tic]	[t/tic] extended to diagrams
The complacent view	YES	NO
Shin's alternative view	YES	YES

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	[t/tic] applied to symbols	[t/tic] applied to diagrams
The complacent view	YES	NO
Shin's alternative view	YES	YES

The two views espouse opposing stance on the issue of whether [t/tic] can be extended to diagrams, but they both consider that [t/tic] is true. In fact, they take this for granted.

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We will show, in the next section, that in philosophy of language, [t/tic] has been challenged. And this could make a difference for how one treats diagrammatic representation.

Identity and Transmission: David Kaplan

In “Words” (1990), Kaplan explains he wants to “contrast two theories of the individuation (one might say, *metaphysics*) of words: the conventional token/type theory [t/tic], which I call the *orthographic* conception, and an alternative theory [tic], which I call the *common currency* conception. I will try slowly to entice you into abandoning the traditional favorite in favor of the new conception.”

The orthographic conception:

[t/tic] The identity of a symbolic representation-token is determined by the type it belongs to.

Kaplan's common currency conception:

[tic] The identity of a symbolic token is determined by the chain of transmission it belongs to.

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In Kaplan's view, one still distinguishes between **particular items** (written inscriptions or verbal utterances) and **word**: two inscriptions may (or may not) be inscriptions of the same word.

But the relationship between words and inscriptions is no longer built on the model of the relationship between a type and a token.

- Two inscriptions are inscriptions of the same word if they belong to the same transmission chain within a given linguistic community (“inscriptions are stages of words, which are the continuants made up of [the] interpersonal stages along with some ... intrapersonal stages”): a word is a historical and social process.
- To decide if two inscriptions are inscriptions of the same word, one has to investigate about the existence of some common transmission chains: Kaplan opposes his naturalistic approach to the Platonist approach underlying the orthographic conception.

Kaplan's view:

"I propose a quite different model according to which utterances and inscriptions are stages of words, which are the continuants made up of these interpersonal stages along with some more mysterious intrapersonal stages. ... This is not, I think, simply another way of doing the metaphysics of types under the old token/type conception, but a quite different conception of the fundamental elements of language. I think of my conception of a word as a **naturalistic** conception."

Kaplan's main objection against [t/tic]:

“It seems to me in many ways that [the orthographic conception] is a sort of updated version of the **Platonic** notion of abstract forms. The eternal, unchanging Platonic forms (shapes, perhaps) are the types, and their physical embodiments, which reflect these abstract forms, are the tokens. I think that the token/type model is the wrong model for the occurrence/word distinction (i.e. the utterance/word distinction or the inscription/word distinction).”

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The application of [t/tic] and [tic] leads to significant discrepancies.

There are cases where **two** inscriptions are considered as belonging to **one** word, according to [tic], and to two words, according to [t/tic].

Kaplan: “On my conception, there is a single word “color” spelled one way in Canada: “c”, “o”, “l” , “o”, “u”, “r” and another way in the United States, “c”, “o”, “l”, “o”, “r”. ... We are, of course, familiar with dialectal variation in pronunciation, but I call to your attention that there is also dialectal variation in spelling. This dialectal variation in spelling of the word “color” for example does not, repeat *not*, make for different words.”

There are cases where **two** inscriptions are considered as belonging to **one** word, according to [t/tic], and to two words, according to [tic].

Kaplan: “My mother’s primary care physician is Dr. Shapiro. He referred her to a specialist, another “Dr. Shapiro” as it happened. My mother reported her gratitude to Dr. Shapiro for sending her to Dr. Shapiro and compared Dr. Shapiro’s virtues to those of Dr. Shapiro in a blithe piece of discourse, clearly oblivious to the homonymy. I was racing to keep up (which I was strangely able to do). But from her point of view, she was quite properly using two different words to refer to two different people. Why should there be a problem?”

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Hawthorne & Lepore (2011) made a mixed assessment of the reception of Kaplan's view in philosophy of language and linguistics. But they agree that Kaplan succeeds to establish that a satisfactory theory of word identity cannot be entirely based on [t/tic].

And this is crucial: Kaplan's explicit aim is not to provide a complete alternative to the orthographic conception, but just to show that the type/token model is not something which is forced on us – it is neither self-evident, nor philosophically neutral.

Last important point. Kaplan does not simply oppose [t/tic] to [tic]. He suggests that the attractiveness of the orthographic conception stems from some **historical facts** about our present use of symbols and words.

The emergence of typography and printing, and its worldwide diffusion, has strongly contributed to the success of the orthographic model. Thanks to printing, we do transmit words and symbols today as the orthographic conception says we do.

So, Kaplan acknowledges the relevance of the orthographic conception. But he relates this relevance to certain facts concerning the way symbols are transmitted today.

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Kaplan: “According to [the orthographic] conception, expressions of the language consist of strings of atoms called ‘letters’, certain strings form words. The letters are abstract entities whose tokens, for the **typesetter**, are **individual pieces of type**. (It’s strange that on this conception, type is a token, but that does seem to be the result.) **You surely know this conception very well**. We were all taught it when we studied the syntax of formal languages (not to mention when we took print shop in high school). It belongs to the formal subject, formal syntax. And its study is the study of an algebra.”

	[t/tic] applied to symbols	[t/tic] applied to diagrams
The complacent view	YES	NO
Shin's alternative view	YES	YES
Kaplan's common currency	NO	

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Identity of Diagrams

In the previous part we have seen how:

- [t/tic] is a shared assumption of both supporters and detractors of the use of diagrams as tools of justification;
- [t/tic] does not adapt itself to diagrams in a simple way, but presents some problems: unique readability etc.
- [t/tic] does not have a unanimous consensus within the philosophers of language;
- Kaplan proposes [tic] as an alternative approach to the occurrence/word distinction

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Goal of this section:

Rather than applying the orthographic approach to diagrams, why don't we attempt to understand them according to the alternative proposed by Kaplan ? What form would a [tic]-theory of diagrams take ?

As we've seen, [tic] establishes a close relationship between identity of words and their transmission: it's by analyzing their history *as continuants* that we can define their identity.

Can we do the same with diagrams ?

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A (very well?) known text: Euclid’s *Elements*

	Propositions A-V-H-H	Diagrams A-V-H-H
Book I	48	48
Book II	14	14
Book III	37	42 (A41)
Book IV	16	16
Book V	25	26 (V25)
Book VI	33	35
Book VII	39	41
Book VIII	27	28
Book IX	36	37
Book X	115	126
Book XI	39	45(A43)
Book XII	18	19
Book XIII	18	21
	465	495/498

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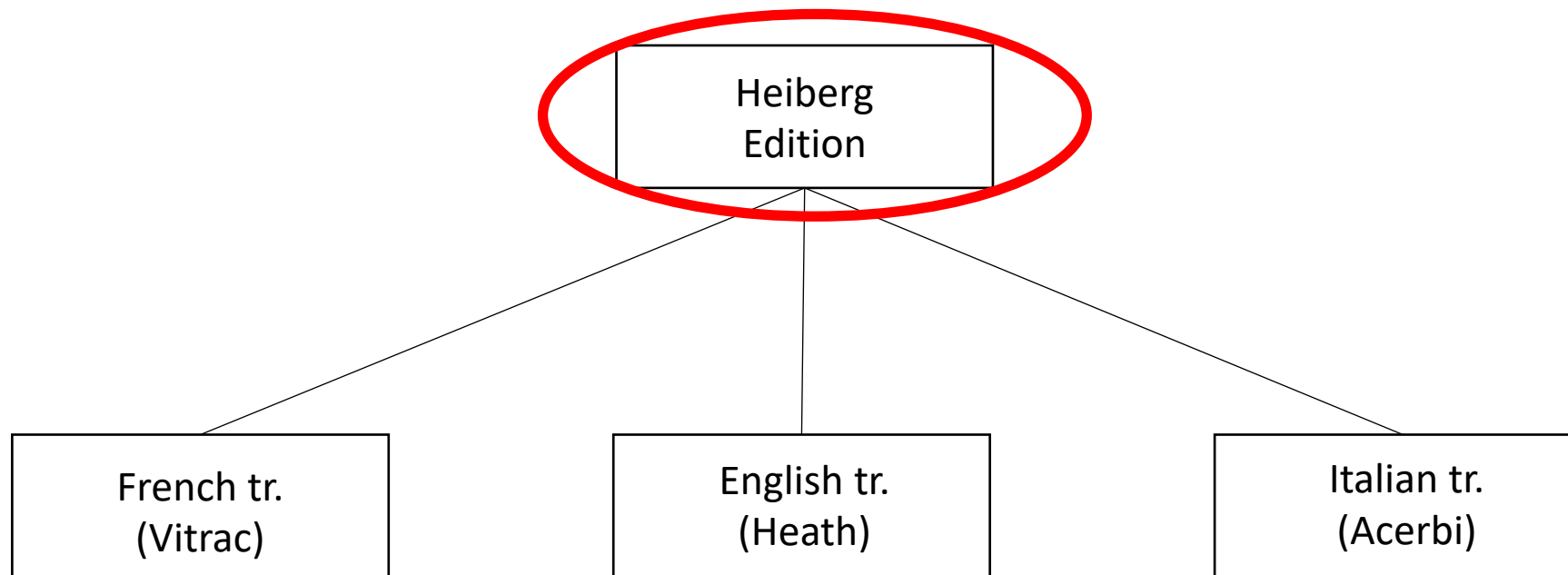
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First (trivial) conclusion:

The *Elements* that we read today in French, English and Italian, from the point of view of transmission, depends directly, **both for text and diagrams**, on the Heiberg edition published between 1883 and 1888.



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"Απαντα ἄρα στερεὰ γωνία ὑπὸ ἐλασσόνων [ἧ] τεσσάρων ὀρθῶν γωνιῶν ἐπιπέδων περιέχεται· ὅπερ εἶδει δεῖξαι.

κβ'.

5 Ἐὰν ὧσι τρεῖς γωνίαι ἐπίπεδοι, ὧν αἱ δύο τῆς λοιπῆς μείζονές εἰσι πάντῃ μεταλαμβανόμεναι, περιέχωσι δὲ αὐτάς ἴσαι εὐθεῖαι, δυνατόν ἐστίν ἐκ τῶν ἐπιξεννυουσῶν τὰς ἴσας εὐθείας τρίγωνον συστήσασθαι.

10 Ἐστῶσαν τρεῖς γωνίαι ἐπίπεδοι αἱ ὑπὸ $ABΓ$, $ΔEZ$, $ΗΘK$, ὧν αἱ δύο τῆς λοιπῆς μείζονές εἰσι πάντῃ μεταλαμβανόμεναι, αἱ μὲν ὑπὸ $ABΓ$, $ΔEZ$ τῆς ὑπὸ $ΗΘK$, αἱ δὲ ὑπὸ $ΔEZ$, $ΗΘK$ τῆς ὑπὸ $ABΓ$, καὶ ἔτι αἱ ὑπὸ $ΗΘK$, $ABΓ$ τῆς ὑπὸ $ΔEZ$, καὶ ἔστωσαν
15 ἴσαι αἱ AB , $ΒΓ$, $ΔE$, EZ , $ΗΘ$, $ΘK$ εὐθεῖαι, καὶ ἐπεξέχθωσαν αἱ $ΑΓ$, $ΔZ$, $ΗK$ · λέγω, ὅτι δυνατόν ἐστίν ἐκ τῶν ἴσων ταῖς $ΑΓ$, $ΔZ$, $ΗK$ τρίγωνον συστήσασθαι, τουτέστιν ὅτι τῶν $ΑΓ$, $ΔZ$, $ΗK$ δύο ὁποιοῦν τῆς λοιπῆς μείζονές εἰσιν.

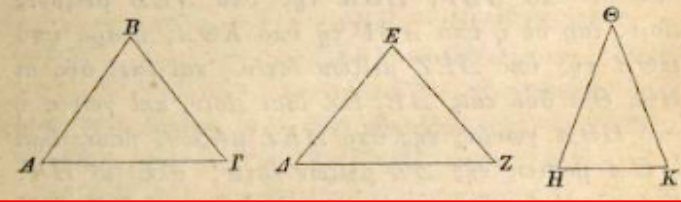
20 Εἰ μὲν οὖν αἱ ὑπὸ $ABΓ$, $ΔEZ$, $ΗΘK$ γωνίαι ἴσαι ἀλλήλαις εἰσίν, φανερόν, ὅτι καὶ τῶν $ΑΓ$, $ΔZ$, $ΗK$ ἴσων γινομένων δυνατόν ἐστίν ἐκ τῶν ἴσων ταῖς $ΑΓ$, $ΔZ$, $ΗK$ τρίγωνον συστήσασθαι. εἰ δὲ οὐ,

1. ἄρα] supra scr. m. 1 P. ὑπό — 3. δεῖξαι]: ~ V.
1. ἧ] postea add. m. 1 P. 7. περιέχουσιν P, περιέχουσι F.
8. Supra ἴσας add. γωνίας m. 2 B, del. m. rec.
εὐθείας] γωνίας εὐθειῶν V. 11. εἰσι] ἔστωσαν BFV et b
(εσ- in ras.). 15. εὐθεῖαι] m. rec. V. 17. συστήσασθαι
P, corr. m. 2. 18. ὅτι] corr. ex τό m. 2 F. 19. μείζονες
V. εἰσι πάντῃ μεταλαμβανόμεναι Theon (BFVb). 21. εἰσι
ἴσαι V. εἰσίν] εἰσι Pbb, comp. F.; om. V. 22. γινομένων
F, γενομένων b.

Ergo omnis¹⁾ angulus solidus planis angulis minoribus, quam sunt quattuor recti, continetur; quod erat demonstrandum.

XXII.

Si tres anguli plani sunt, quorum duo reliquo maiores sunt quoquo modo coniuncti, et eos aequales



continent rectae, fieri potest, ut ex rectis aequales rectas coniungentibus triangulus construatur.

Sint tres anguli plani $ABΓ$, $ΔEZ$, $ΗΘK$, quorum duo reliquo maiores sunt quoquo modo coniuncti,
 $ABΓ + ΔEZ > ΗΘK$, $ΔEZ + ΗΘK > ABΓ$,

$$ΗΘK + ABΓ > ΔEZ,$$

et sit $AB = BΓ = ΔE = EZ = ΗΘ = ΘK$, et ducantur $ΑΓ$, $ΔZ$, $ΗK$. dico, fieri posse, ut ex rectis aequalibus rectis $ΑΓ$, $ΔZ$, $ΗK$ triangulus construatur, hoc est, rectarum $ΑΓ$, $ΔZ$, $ΗK$ duas quaslibet reliqua maiores esse.

iam si anguli $ABΓ$, $ΔEZ$, $ΗΘK$ inter se aequales sunt, manifestum est, cum etiam $ΑΓ$, $ΔZ$, $ΗK$ aequales sint [I, 4], fieri posse, ut ex rectis aequalibus rectis $ΑΓ$, $ΔZ$, $ΗK$ triangulus construatur. sin minus, in-

1) Nam in angulis solidis, qui plus quam tribus planis angulis continentur, similiter ratiocinandum est.

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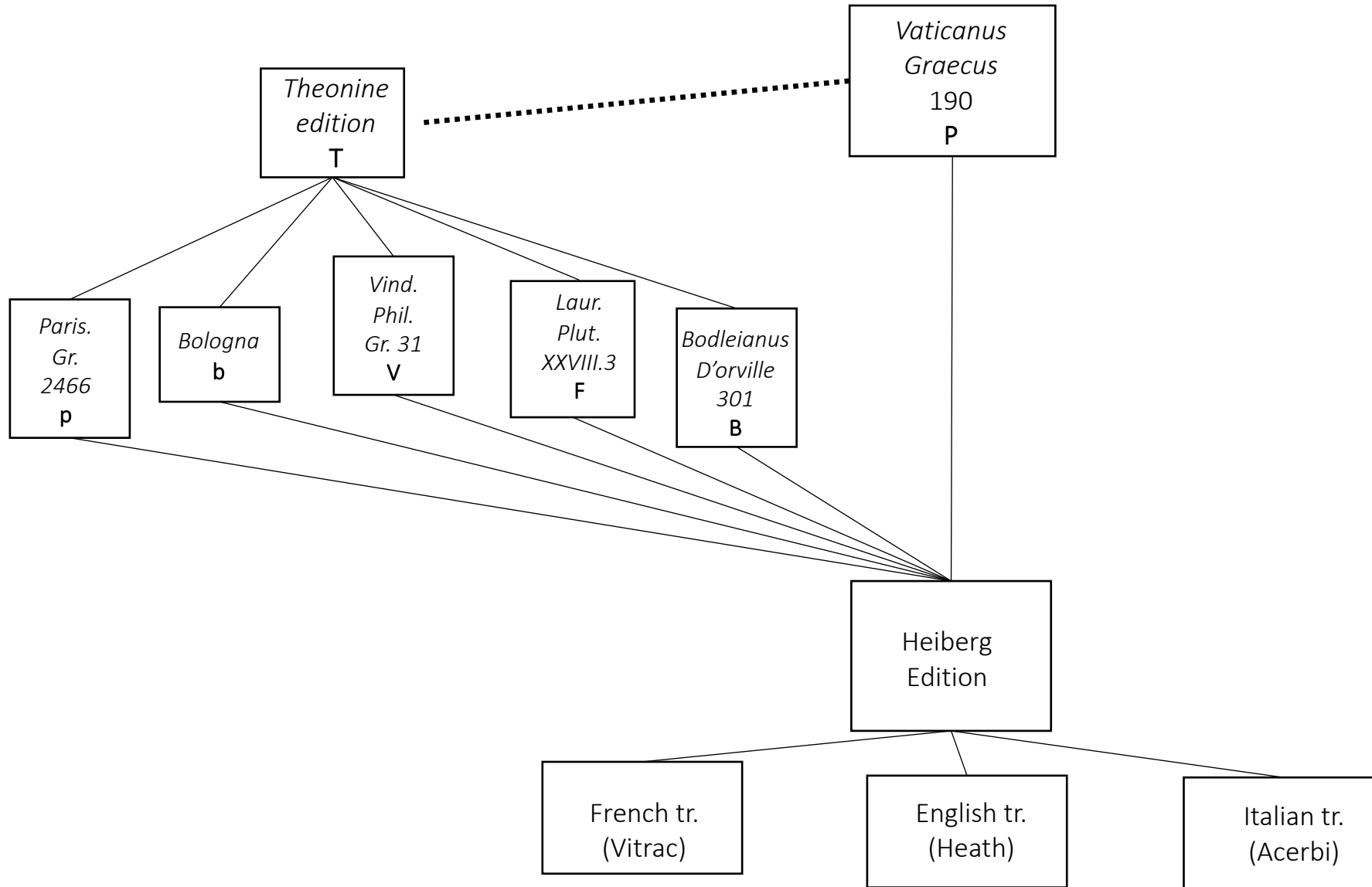
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‘Lachmann method’ (*textkritik*)

Reconstruction of the «*text*» on the evidence of the surviving manuscripts:

1. *Recensio*
2. *Emendatio*

Heiberg text collation



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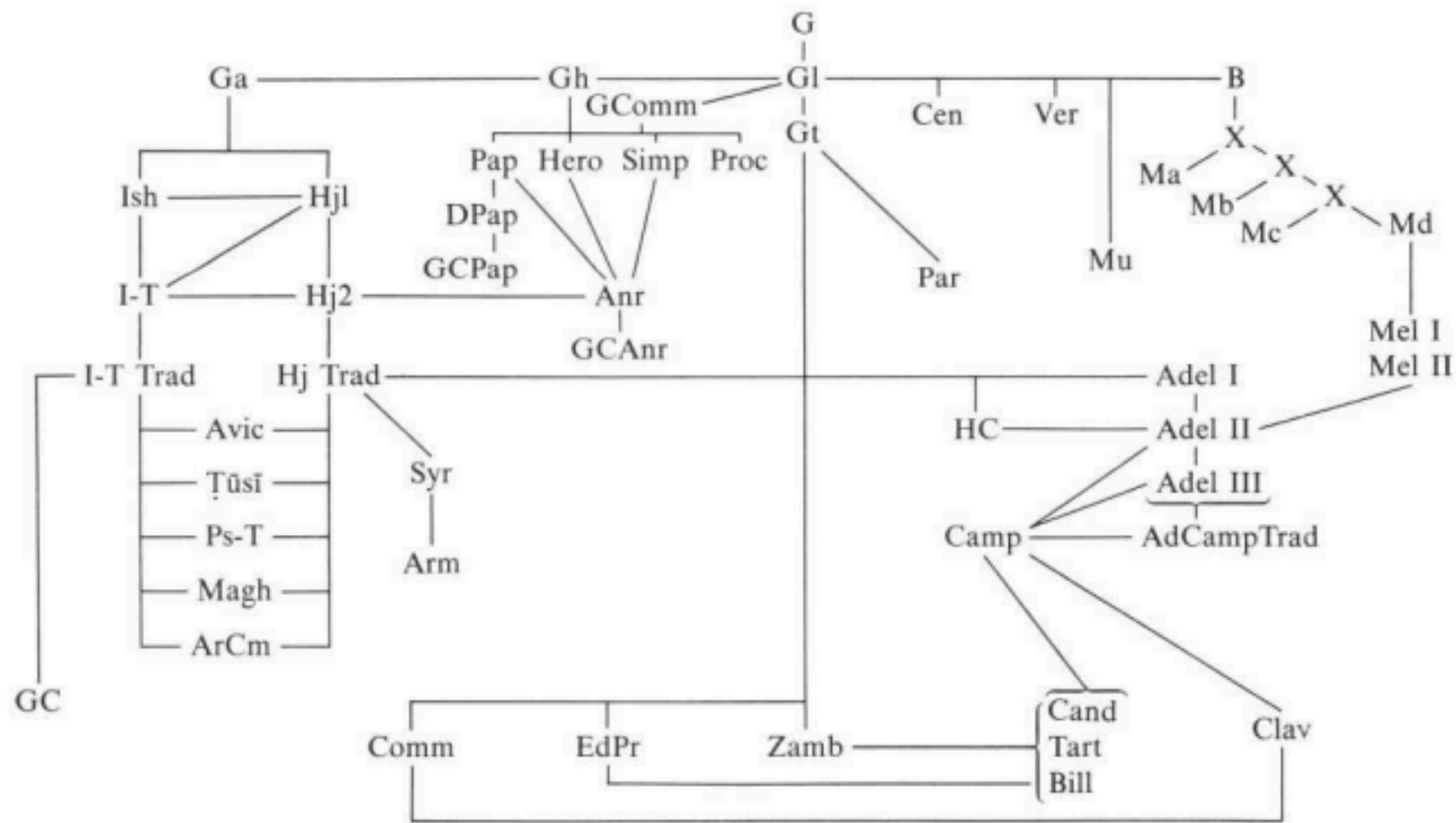
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The text of Euclid's *Elements* today



(Murdoch, *DSB* IV,458)

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This quick analysis leads us to conclude, concerning Euclid text, that:

(E-Text)

We know relatively well some part of the history of the transmission of the text of the *Elements* and we are able to grasp the chains of transmission by which it has come down to us.

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"Απασα ἄρα στερεὰ γωνία ὑπὸ ἐλασσόνων [ῆ] τεσσάρων ὀρθῶν γωνιῶν ἐπιπέδων περιέχεται· ὅπερ εἶδει δεῖξαι.

κβ'.

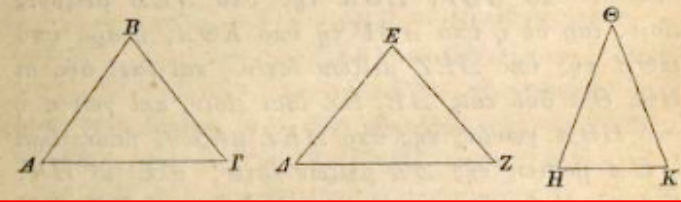
- 5 Ἐὰν ὧσι τρεῖς γωνίαι ἐπίπεδοι, ὧν αἱ δύο τῆς λοιπῆς μείζονές εἰσι πάντῃ μεταλαμβανόμεναι, περιέχωσι δὲ αὐτὰς ἴσαι εὐθεῖαι, δυνατόν ἐστίν ἐκ τῶν ἐπιξεννυουσῶν τὰς ἴσας εὐθείας τρίγωνον συστήσασθαι.
- 10 Ἐστῶσαν τρεῖς γωνίαι ἐπίπεδοι αἱ ὑπὸ $AB\Gamma$, ΔEZ , $H\Theta K$, ὧν αἱ δύο τῆς λοιπῆς μείζονές εἰσι πάντῃ μεταλαμβανόμεναι, αἱ μὲν ὑπὸ $AB\Gamma$, ΔEZ τῆς ὑπὸ $H\Theta K$, αἱ δὲ ὑπὸ ΔEZ , $H\Theta K$ τῆς ὑπὸ $AB\Gamma$, καὶ ἔτι αἱ ὑπὸ $H\Theta K$, $AB\Gamma$ τῆς ὑπὸ ΔEZ , καὶ ἐστῶσαν
- 15 ἴσαι αἱ AB , $B\Gamma$, ΔE , EZ , $H\Theta$, ΘK εὐθεῖαι, καὶ ἐπεξέχθωσαν αἱ $A\Gamma$, ΔZ , HK · λέγω, ὅτι δυνατόν ἐστίν ἐκ τῶν ἴσων ταῖς $A\Gamma$, ΔZ , HK τρίγωνον συστήσασθαι, τουτέστιν ὅτι τῶν $A\Gamma$, ΔZ , HK δύο ὁποιοῦν τῆς λοιπῆς μείζονές εἰσιν.
- 20 Εἰ μὲν οὖν αἱ ὑπὸ $AB\Gamma$, ΔEZ , $H\Theta K$ γωνίαι ἴσαι ἀλλήλαις εἰσίν, φανερόν, ὅτι καὶ τῶν $A\Gamma$, ΔZ , HK ἴσων γινομένων δυνατόν ἐστίν ἐκ τῶν ἴσων ταῖς $A\Gamma$, ΔZ , HK τρίγωνον συστήσασθαι. εἰ δὲ οὐ,

1. ἄρα] supra scr. m. 1 P. ὑπὸ — 3. δεῖξαι]: ~ V.
1. ῆ] postea add. m. 1 P. 7. περιέχουσιν P, περιέχονσι F.
8. Supra ἴσας add. γωνίας m. 2 B, del. m. rec.
εὐθείας] γωνίας εὐθειῶν V. 11. εἰσι] ἐστῶσαν BFV et b (εσ- in ras.). 15. εὐθεῖαι] m. rec. V. 17. συστήσασθαι P, corr. m. 2. 18. ὅτι] corr. ex τό m. 2 F. 19. μείζονες V. εἰσι πάντῃ μεταλαμβανόμεναι Theon (BFVb). 21. εἰσι ἴσαι V. εἰσίν] εἰσι Pbb, comp. F.; om. V. 22. γινομένων F, γενομένων b.

Ergo omnis¹⁾ angulus solidus planis angulis minoribus, quam sunt quattuor recti, continetur; quod erat demonstrandum.

XXII.

Si tres anguli plani sunt, quorum duo reliquo maiores sunt quoquo modo coniuncti, et eos aequales



continent rectae, fieri potest, ut ex rectis aequales rectas coniungentibus triangulus construatur.

Sint tres anguli plani $AB\Gamma$, ΔEZ , $H\Theta K$, quorum duo reliquo maiores sunt quoquo modo coniuncti, $AB\Gamma + \Delta EZ > H\Theta K$, $\Delta EZ + H\Theta K > AB\Gamma$,

$$H\Theta K + AB\Gamma > \Delta EZ,$$

et sit $AB = B\Gamma = \Delta E = EZ = H\Theta = \Theta K$, et ducantur $A\Gamma$, ΔZ , HK . dico, fieri posse, ut ex rectis aequalibus rectis $A\Gamma$, ΔZ , HK triangulus construatur, hoc est, rectarum $A\Gamma$, ΔZ , HK duas quaslibet reliqua maiores esse.

iam si anguli $AB\Gamma$, ΔEZ , $H\Theta K$ inter se aequales sunt, manifestum est, cum etiam $A\Gamma$, ΔZ , HK aequales sint [I, 4], fieri posse, ut ex rectis aequalibus rectis $A\Gamma$, ΔZ , HK triangulus construatur. sin minus, in-

1) Nam in angulis solidis, qui plus quam tribus planis angulis continentur, similiter ratiocinandum est.

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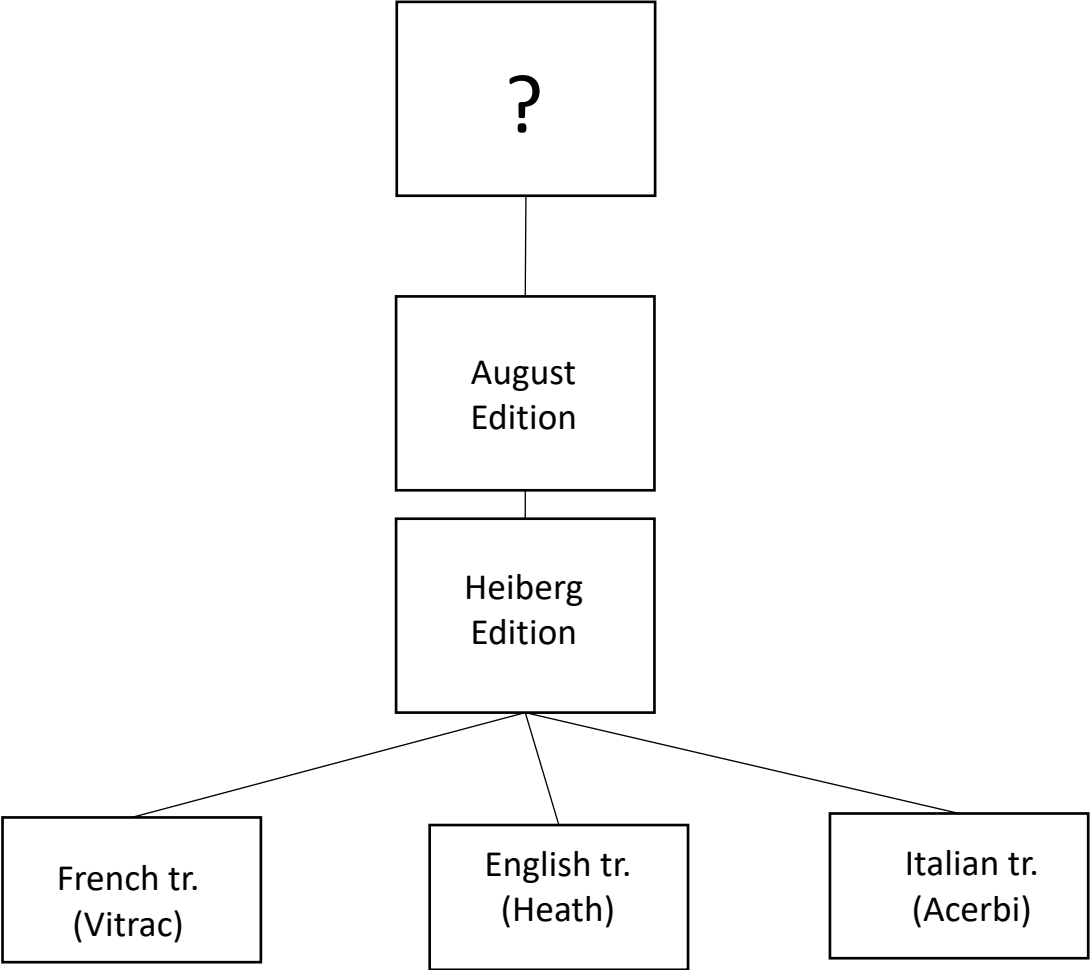
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Beyond Heiberg: Philology of diagrams

K. Saito, “A Preliminary Study in the Critical Assessment of Diagrams in Greek Mathematical Works” (2006), “Reproduced Diagrams of the *Elements*” (2008), “The diagrams of Book VI of the *Elements* in two Greek manuscripts” (2008), “The Diagrams of Book IV of the *Elements* in Greek Manuscripts” (2008), “The Diagrams of Book II and III of the *Elements* in Greek Manuscripts” (2011)

K. Saito et N. Sidoli, “Diagrams and Arguments in Ancient Greek Mathematics : Lessons Drawn from Comparisons of the Manuscript Diagrams with those in Modern Critical Editions” (2012)

A. Malet, “Euclid’s Swan Song : Euclid’s *Elements* in Early Modern Europe” (2012)

E. Lee, “Let the Diagram Speak : Compass Arcs and Visual Auxiliaries in Printed Diagrams of Euclid’s *Elements*” (2018)

G. De Young, “Diagrams in the Arabic Euclidean Tradition : A Preliminary Assessment” (2005)
“Mathematical Diagrams from Manuscript to Print : Examples from the Arabic Euclidean Transmission” (2012)

Other works: Spherics (Sidoli), Apollonius’ Conics (Decorps), Archimedean corpus (Netz)

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(E-Text)

We know relatively well some part of the history of the transmission of the text of the *Elements* and we are able to grasp the chains of transmission by which it has come down to us.

(E-Diagrams)

Thanks to the work done during the last decades we better know part of the history of the transmission of Euclidean diagrams and we are able to grasp the chains of transmission by which they have come down to us.

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Can we extend [tic] to diagrams ?

Our answer is:

Philology has already begun to do it !

As we have seen, Kaplan establishes a close relationship between identity of words and their transmission. The new philology extends the methodology used to study textual transmission in Euclid's *Elements* to the study of diagram transmission. This is exactly what we were looking for.

What do we learn about transmission of diagrams from these philological studies ?

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Copying and Printing

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Thanks to the work done during the last decades we better know *part* of the history of the transmission of Euclidean diagrams and we are able to grasp the chains of transmission by which they have come down to us.

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(E-Text)

We know relatively well some part of the history of the transmission of the text of the *Elements* and we are able to grasp the chains of transmission by which it has come down to us.

continuity: medieval manuscripts – printed editions – Heiberg edition – our text

(E-Diagrams)

Thanks to the work done during the last decades we better know *part* of the history of the transmission of Euclidean diagrams and we are able to grasp the chains of transmission by which they have come down to us.

Discontinuity: medieval manuscripts // printed editions – Heiberg edition – our diagrams

Printing revolution changed the way by which diagrams were conceived and produced

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“Observing the manuscript diagrams makes it clear that medieval scribes rarely, if ever, attempted actual construction with a compass and a straightedge. Rather, the **diagrams were copied by eyeballing approximate diagrams that were hand-drawn in previous manuscripts**. In this way, diagrams in medieval copies of the *Elements* were drawn not by the construction but through the transcription [...]. The transcription of diagrams came to an end with the advent of the printing press, **by which printers introduced a new type of drawing, that is, construction of diagrams through the explicit use of tools such as compass and straight edges.**” (Lee 2018, 78)

Manuscript diagrams ---- Copied

Printed diagrams ----- Procedural

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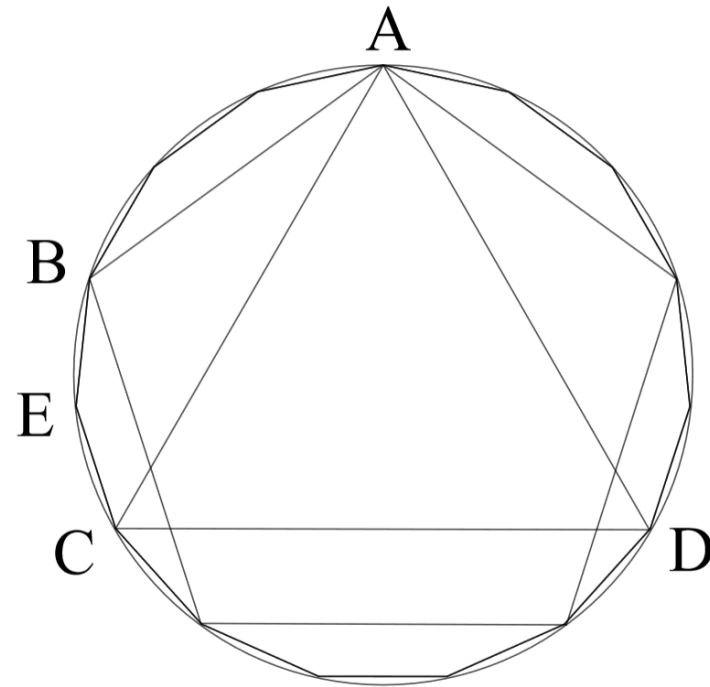
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An exemple

El. IV.16

To inscribe an equilateral and equiangular fifteen-angled figure in a given circle.



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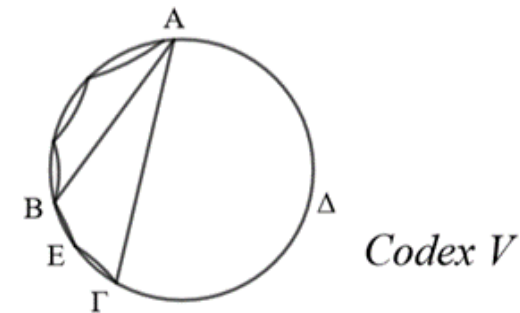
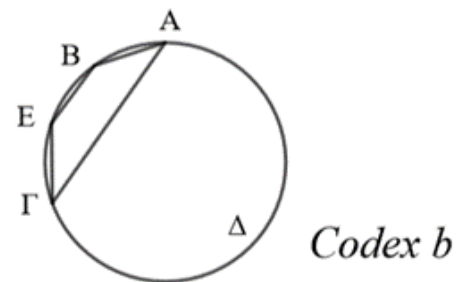
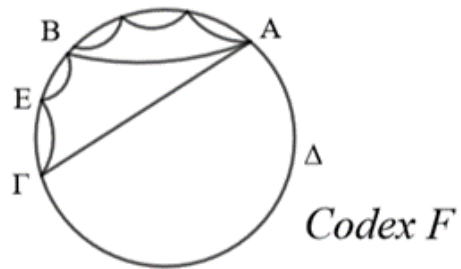
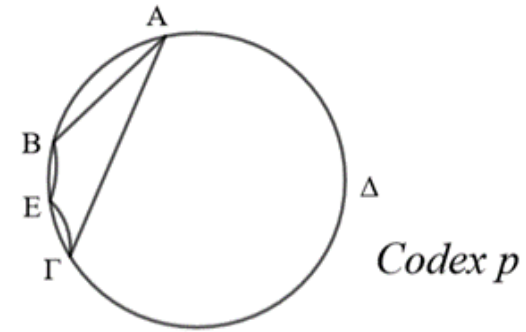
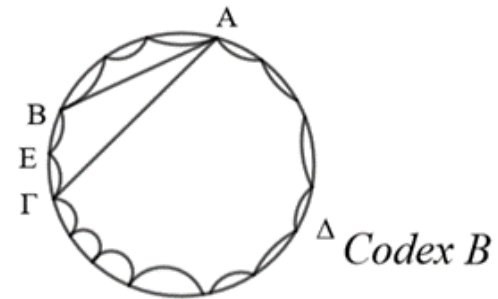
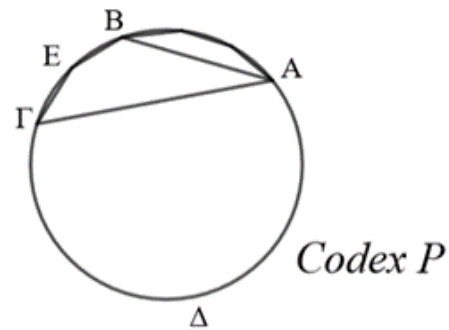
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El. IV.16: Manuscript tradition



(Saito 2008, 23 *mod.*)

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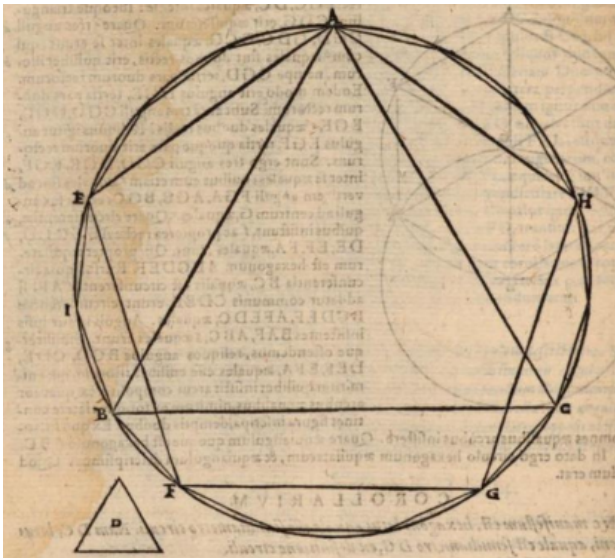
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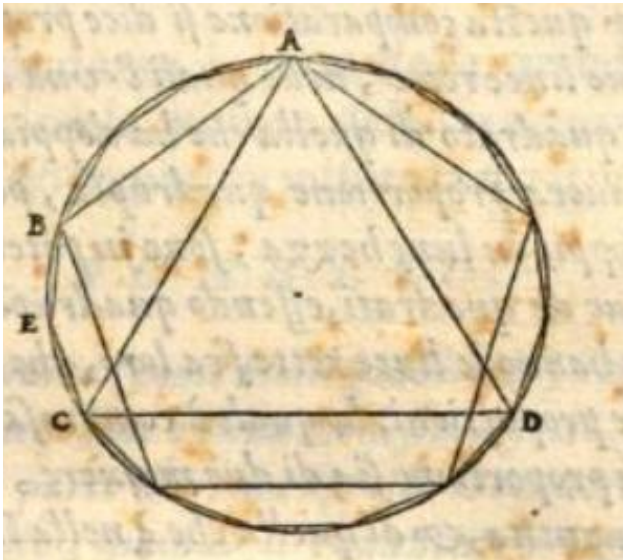
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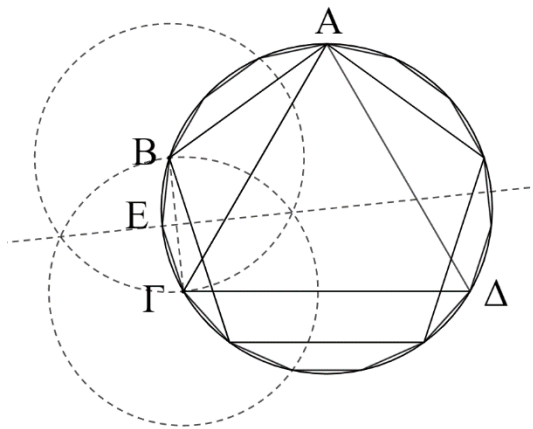
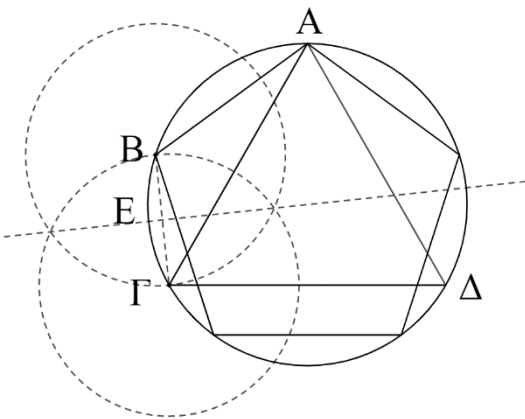
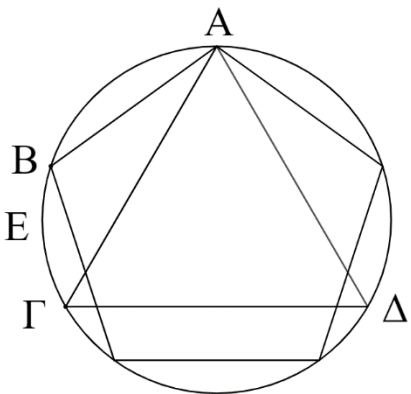
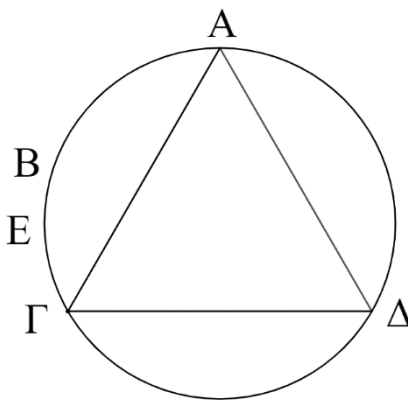
El. IV.16: Early printed transmission



Clavius (1574, 186v)



Commandinus (1575, 61r)



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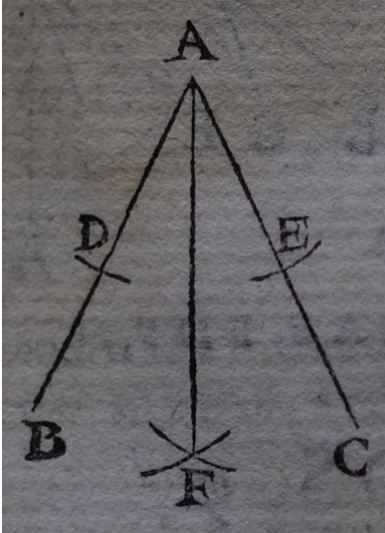
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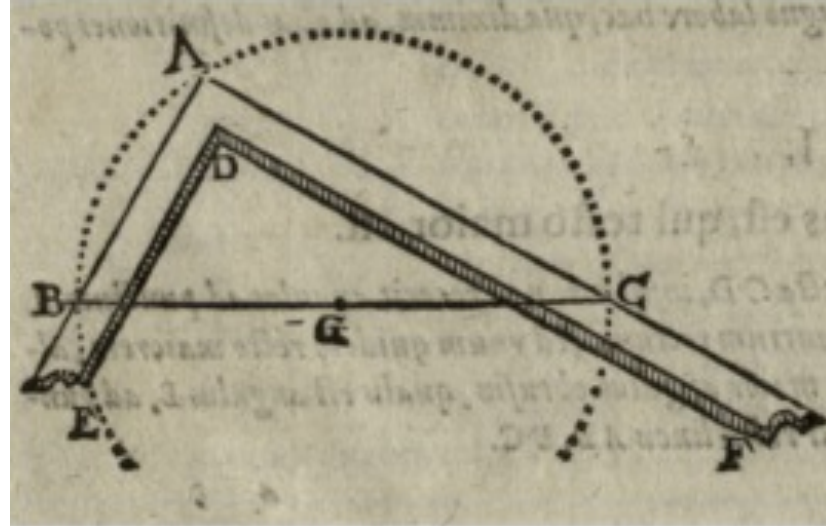
Conclusion

This transition from “copied diagrams” to “procedural diagrams”, illustrated by *El.* IV.16, reflects a **general tendency**:

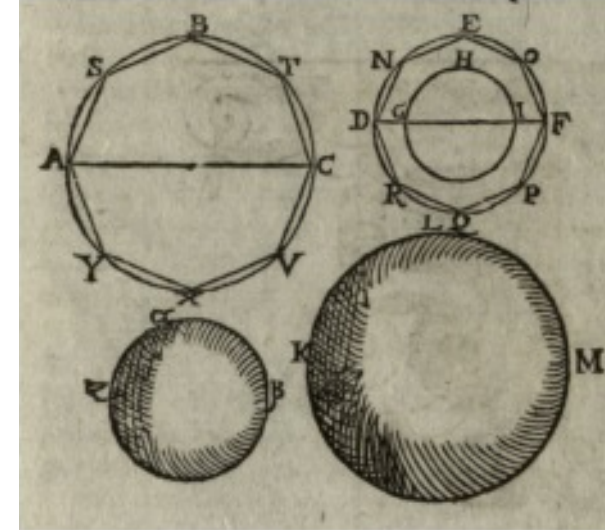
“Diagram makers began to introduce more efficient and practical methods of construction. As a result, new forms of diagrams emerged together with compass arcs, partial constructions, and more frequent use of the circle, all of which reflected the active use of tools” (Lee 2018)



Compass arcs



Geometric Tools
and dotted lines



Shadows

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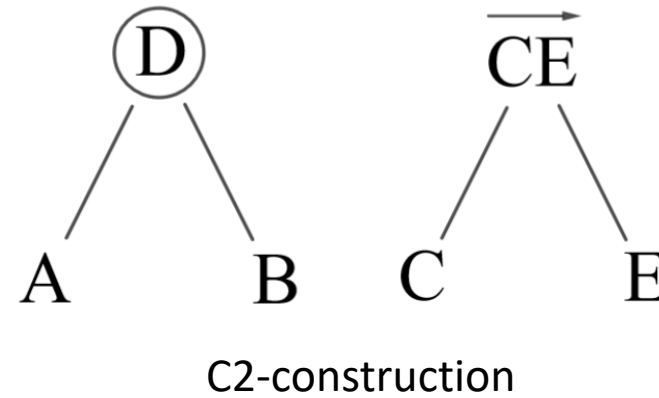
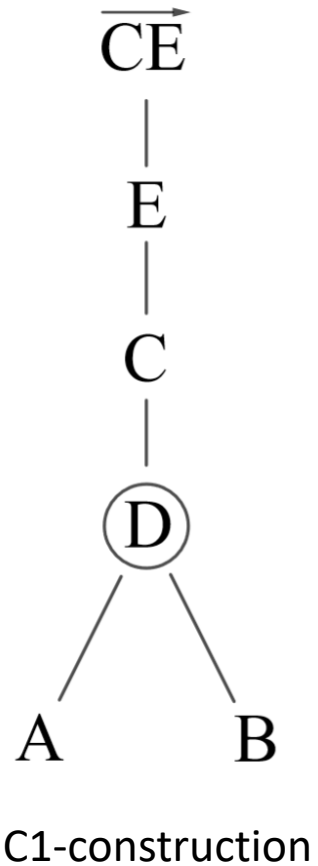
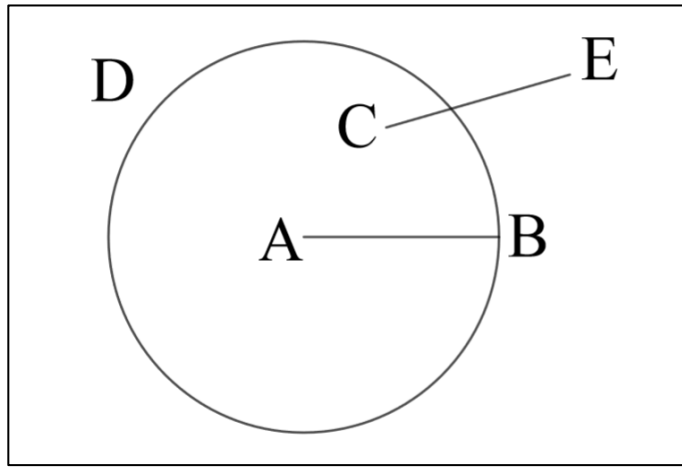
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The Renaissance procedural diagrams are a perfect candidate for a [t/tic] approach like that one developed by Mumma:



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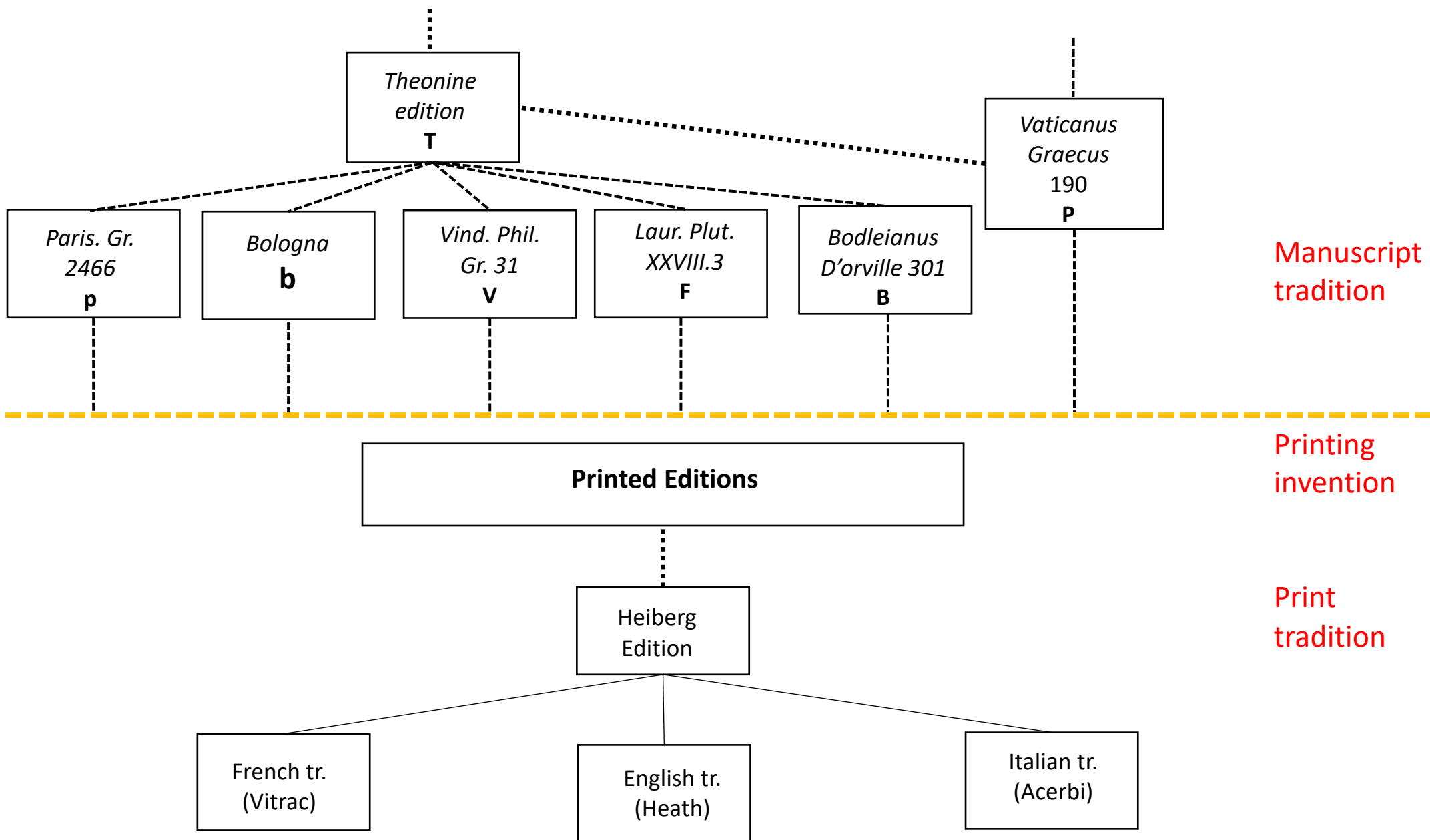
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Coming back to Kaplan...

In his account, Kaplan does not entirely reject [t/tic]. He suggests to re-contextualize it: according to him, [t/tic] is linked to a particular mode of transmission, the one that emerged with the development of the printing techniques.

“According to [the orthographic] conception, expressions of the language consist of strings of atoms called ‘letters’, certain strings form words. The letters are abstract entities whose tokens, for the **typesetter**, are **individual pieces of type**. You surely know this conception very well. We were all taught it when we studied the syntax of formal languages (not to mention when we took print shop in high school). It belongs to the formal subject, formal syntax. And its study is the study of an algebra.”

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Coming back to Kaplan...

The idea that Euclidean diagrams are the results of a regulated construction procedure is applicable only starting from Renaissance diagrams, when the old division of tasks – copy of the text, copy of the diagrams - disappeared to leave room for a more integrative perspective.

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Conclusion 1

The discussion about diagrams is usually based on the idea that there is a contrast between symbolic and diagrammatic representations.

We don't want to deny there is such a contrast. Our point is simply that the way we conceive symbols has an impact on the way the discussion about diagram is framed.

Kaplan's "metaphysics of symbol" is perhaps untenable. The idea to extend Kaplan's metaphysics to diagram is perhaps also untenable.

But nevertheless, we think there is still something left in the journey we have sketched out : **the philosophy of diagrams should not be disconnected from the philosophy of language.**

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Conclusion 2

[tic] establishes a close relationship between identity of words and their transmission: it's by analyzing their history *as continuants* that we can define their identity. Can we extend [tic] to diagrams ?

Our answer is: **philology already does it !**

- If such an extension is possible, it cannot be made a priori. To characterize how diagrams have been identified and how their identity differs from that of symbols, there is no other way than to make the history of their transmission.
- But, conversely, putting these historical studies in the broader perspective provided by Kaplan allows us to see where lies their philosophical interest: they do tell us something about what diagrams are.

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Conclusion 3

A perspective

The parallel reading of Kaplan and the literature on the philology of diagrams to which we have devoted ourselves seems to suggest that the emergence of the printing techniques constitutes a turning point in the transmission of diagrams.

In our opinion, there is an important field of research to be explored here

Greek geometrical analysis,
Astronomical tables (Ptolemy)

Thanks